

# Fg Fg

# **CHAPTER 8**

# Fluid Mechanics

Kayakers know that if their weight  $(F_g)$  exceeds the upward, buoyant force  $(F_B)$  that causes them to float, they are sunk—literally! For an object, such as a kayak, that is immersed in a fluid, buoyant force equals the weight of the fluid that the object displaces. Buoyant force causes a kayak to pop to the surface after a plunge down a waterfall.



In this chapter, you will learn about buoyant force, fluid pressure, and the basic equations that govern the behavior of fluids. This chapter will also introduce moving fluids and the continuity equation.

# WHY IT MATTERS

Many kinds of hydraulic devices, such as the brakes in a car and the lifts that move heavy equipment, make use of the properties of fluids. An understanding of the properties of fluids is needed to design such devices.

# **CHAPTER PREVIEW**

- 1 Fluids and Buoyant Force Defining a Fluid Density and Buoyant Force
- 2 Fluid Pressure Pressure
- 3 Fluids in Motion Fluid Flow Principles of Fluid Flow



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# **SECTION 1**

#### **SECTION OBJECTIVES**

- Define a fluid.
- Distinguish a gas from a liquid.
- Determine the magnitude of the buoyant force exerted on a floating object or a submerged object.
- Explain why some objects float and some objects sink.

#### fluid

a nonsolid state of matter in which the atoms or molecules are free to move past each other, as in a gas or a liquid



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#### **ADVANCED TOPICS**

See "Properties of Gases" in **Appendix J: Advanced Topics** to learn more about how gases behave.

# Fluids and Buoyant Force

# **DEFINING A FLUID**

Matter is normally classified as being in one of three states—solid, liquid, or gaseous. Up to this point, this book's discussion of motion and the causes of motion has dealt primarily with the behavior of solid objects. This chapter concerns the mechanics of liquids and gases.

**Figure 1(a)** is a photo of a liquid; **Figure 1(b)** shows an example of a gas. Pause for a moment and see if you can identify a common trait between them. One property they have in common is the ability to flow and to alter their shape in the process. Materials that exhibit these properties are called **fluids.** Solid objects are not considered to be fluids because they cannot flow and therefore have a definite shape.

#### Liquids have a definite volume; gases do not

Even though both gases and liquids are fluids, there is a difference between them: one has a definite volume, and the other does not. Liquids, like solids, have a definite volume, but unlike solids, they do not have a definite shape. Imagine filling the tank of a lawn mower with gasoline. The gasoline, a liquid, changes its shape from that of its original container to that of the tank. If there is a gallon of gasoline in the container before you pour, there will be a gallon in the tank after you pour. Gases, on the other hand, have neither a definite volume nor a definite shape. When a gas is poured from a smaller container into a larger one, the gas not only changes its shape to fit the new container but also spreads out and changes its volume within the container.





## Figure 1

Both (a) liquids and (b) gases are considered fluids because they can flow and change shape.

# **DENSITY AND BUOYANT FORCE**

Have you ever felt confined in a crowded elevator? You probably felt that way because there were too many people in the elevator for the amount of space available. In other words, the *density* of people was too high. In general, density is a measure of a quantity in a given space. The quantity can be anything from people or trees to mass or energy.

#### Mass density is mass per unit volume of a substance

When the word *density* is used to describe a fluid, what is really being measured is the fluid's **mass density**. Mass density is the mass per unit volume of a substance. It is often represented by the Greek letter  $\rho$  (*rho*).



The SI unit of mass density is kilograms per cubic meter  $(kg/m^3)$ . In this book we will follow the convention of using the word *density* to refer to *mass density*. **Table 1** lists the densities of some fluids and a few important solids.

Solids and liquids tend to be almost incompressible, meaning that their density changes very little with changes in pressure. Thus, the densities listed in **Table 1** for solids and liquids are approximately independent of pressure. Gases, on the other hand, are compressible and can have densities over a wide range of values. Thus, there is not a standard density for a gas, as there is for solids and liquids. The densities listed for gases in **Table 1** are the values of the density at a stated temperature and pressure. For deviations of temperature and pressure from these values, the density of the gas will vary significantly.

#### Buoyant forces can keep objects afloat

Have you ever wondered why things feel lighter underwater than they do in air? The reason is that a fluid exerts an upward force on objects that are partially or completely submerged in it. This upward force is called a **buoyant force.** If you have ever rested on an air mattress in a swimming pool, you have experienced a buoyant force. The buoyant force kept you and the mattress afloat.

Because the buoyant force acts in a direction opposite the force of gravity, the net force acting on an object submerged in a fluid, such as water, is smaller than the object's weight. Thus, the object appears to weigh less in water than it does in air. The weight of an object immersed in a fluid is the object's *apparent weight*. In the case of a heavy object, such as a brick, its apparent weight is less in water than its actual weight is in air, but it may still sink in water because the buoyant force is not enough to keep it afloat.

#### mass density

the concentration of matter of an object, measured as the mass per unit volume of a substance

#### Table 1 Densities of Some Common Substances\*

Substance	ho (kg/m <sup>3</sup> )	
Hydrogen	0.0899	
Helium	0.179	
Steam (100°C)	0.598	
Air	1.29	
Oxygen	1.43	
Carbon dioxide	1.98	
Ethanol	$0.806  imes 10^3$	
lce	$0.917 \times 10^3$	
Fresh water (4°C)	$1.00  imes 10^3$	
Sea water (15°C)	$1.025\times10^3$	
Iron	$7.86 \times 10^3$	
Mercury	$13.6 \times 10^3$	
Gold	$19.3\times10^3$	

\*All densities are measured at 0°C and 1 atm unless otherwise noted.

#### buoyant force

the upward force exerted by a liquid on an object immersed in or floating on the liquid

#### Figure 2

(a) A brick is being lowered into a container of water. (b) The brick displaces water, causing the water to flow into a smaller container.
(c) When the brick is completely submerged, the volume of the displaced water (d) is equal to the volume of the brick.



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# **Did you know?**

Archimedes was a Greek mathematician who was born in Syracuse, a city on the island of Sicily. According to legend, the king of Syracuse suspected that a certain golden crown was not pure gold. While bathing, Archimedes figured out how to test the crown's authenticity when he discovered the buoyancy principle. He is reported to have then exclaimed, "Eureka!" meaning "I've found it!"





#### Archimedes' principle describes the magnitude of a buoyant force

Imagine that you submerge a brick in a container of water, as shown in **Figure 2.** A spout on the side of the container at the water's surface allows water to flow out of the container. As the brick sinks, the water level rises and water flows through the spout into a smaller container. The total volume of water that collects in the smaller container is the *displaced volume* of water from the large container. The displaced volume of water is equal to the volume of the portion of the brick that is underwater.

The magnitude of the buoyant force acting on the brick at any given time can be calculated by using a rule known as *Archimedes' principle*. This principle can be stated as follows: *Any object completely or partially submerged in a fluid experiences an upward buoyant force equal in magnitude to the weight of the fluid displaced by the object*. Everyone has experienced Archimedes' principle. For example, recall that it is relatively easy to lift someone if you are both standing in a swimming pool, even if lifting that same person on dry land would be difficult.

Using  $m_f$  to represent the mass of the displaced fluid, Archimedes' principle can be written symbolically as follows:

#### **BUOYANT FORCE**

 $F_B = F_g(displaced fluid) = m_f g$ 

magnitude of buoyant force = weight of fluid displaced

Whether an object will float or sink depends on the net force acting on it. This net force is the object's apparent weight and can be calculated as follows:

$$F_{net} = F_B - F_g(object)$$

Now we can apply Archimedes' principle, using  $m_o$  to represent the mass of the submerged object.

$$F_{net} = m_f g - m_o g$$

Remember that  $m = \rho V$ , so the expression can be rewritten as follows:

$$F_{net} = (\rho_f V_f - \rho_o V_o)g$$

Note that in this expression, the fluid quantities refer to the *displaced* fluid.

#### For a floating object, the buoyant force equals the object's weight

Imagine a cargo-filled raft floating on a lake. There are two forces acting on the raft and its cargo: the downward force of gravity and the upward buoyant force of the water. Because the raft is floating in the water, the raft is in equilibrium and the two forces are balanced, as shown in **Figure 3.** For floating objects, the buoyant force and the weight of the object are equal in magnitude.

#### **BUOYANT FORCE ON FLOATING OBJECTS**

 $F_B = F_g(object) = m_o g$ buoyant force = weight of floating object

Notice that Archimedes' principle is not required to find the buoyant force on a floating object if the weight of the object is known.

#### The apparent weight of a submerged object depends on density

Imagine that a hole is accidentally punched in the raft shown in **Figure 3** and that the raft begins to sink. The cargo and raft eventually sink below the water's surface, as shown in **Figure 4**. The net force on the raft and cargo is the vector sum of the buoyant force and the weight of the raft and cargo. As the volume of the raft decreases, the volume of water displaced by the raft and cargo also decreases, as does the magnitude of the buoyant force. This can be written by using the expression for the net force:

$$F_{net} = (\rho_f V_f - \rho_o V_o)g$$

Because the raft and cargo are completely submerged,  $V_f$  and  $V_o$  are equal:

$$F_{net} = (\rho_f - \rho_o) V_{\xi}$$

Notice that both the direction and the magnitude of the net force depend on the difference between the density of the object and the density of the fluid in which it is immersed. If the object's density is greater than the fluid density, the net force is negative (downward) and the object sinks. If the object's density is less than the fluid density, the net force is positive (upward) and the object rises to the surface and floats. If the densities are the same, the object hangs suspended underwater.

A simple relationship between the weight of a submerged object and the buoyant force on the object can be found by considering their ratio as follows:

$$\frac{F_g(object)}{F_B} = \frac{\rho_o \forall g}{\rho_f \forall g}$$
$$\frac{F_g(object)}{F_B} = \frac{\rho_o}{\rho_f}$$

This last expression is often useful in solving buoyancy problems.



**Figure 3** The raft and cargo are floating because their weight and the buoyant force are balanced.



#### Figure 4 The raft and cargo sink because their density is greater than the density of water.



#### SAMPLE PROBLEM A

#### **Buoyant Force**

#### **PROBLEM**

A bargain hunter purchases a "gold" crown at a flea market. After she gets home, she hangs the crown from a scale and finds its weight to be 7.84 N. She then weighs the crown while it is immersed in water, and the scale reads 6.86 N. Is the crown made of pure gold? Explain.



#### **2.** PLAN Choose an equation or situation:

Because the object is completely submerged, consider the ratio of the weight to the buoyant force.

$$F_g - F_B =$$
apparent weight  $\frac{F_g}{P_o} - \frac{\rho_o}{P_o}$ 

 $F_B \rho_f$ 

$$F_B = F_g - (\text{apparent weight})$$
$$\rho_o = \frac{F_g}{F_B} \rho_f$$

#### **3.** CALCULATE Substitute the values into the equation and solve:

$$F_B = 7.84 \text{ N} - 6.86 \text{ N} = 0.98 \text{ N}$$

$$\rho_o = \frac{F_g}{F_B} \rho_f = \frac{7.84 \text{ N}}{0.98 \text{ N}} (1.00 \times 10^3 \text{ kg/m}^3)$$

$$\rho_o = 8.0 \times 10^3 \text{ kg/m}^3$$

**4. EVALUATE** From **Table 1**, the density of gold is  $19.3 \times 10^3$  kg/m<sup>3</sup>. Because  $8.0 \times 10^3$  kg/m<sup>3</sup> <  $19.3 \times 10^3$  kg/m<sup>3</sup>, the crown cannot be pure gold.

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#### **Buoyant Force**

- **1.** A piece of metal weighs 50.0 N in air, 36.0 N in water, and 41.0 N in an unknown liquid. Find the densities of the following:
  - **a.** the metal
  - **b.** the unknown liquid
- **2.** A 2.8 kg rectangular air mattress is 2.00 m long, 0.500 m wide, and 0.100 m thick. What mass can it support in water before sinking?
- **3.** A ferry boat is 4.0 m wide and 6.0 m long. When a truck pulls onto it, the boat sinks 4.00 cm in the water. What is the weight of the truck?
- **4.** An empty rubber balloon has a mass of 0.0120 kg. The balloon is filled with helium at 0°C, 1 atm pressure, and a density of 0.179 kg/m<sup>3</sup>. The filled balloon has a radius of 0.500 m.
  - **a.** What is the magnitude of the buoyant force acting on the balloon? (Hint: See **Table 1** for the density of air.)
  - **b.** What is the magnitude of the net force acting on the balloon?

# **SECTION REVIEW**

- **1.** What is the difference between a solid and a fluid? What is the difference between a gas and a liquid?
- 2. Which of the following objects will float in a tub of mercury?
  - a. a solid gold bead
  - **b.** an ice cube
  - c. an iron bolt
  - d. 5 mL of water
- **3.** A 650 kg weather balloon is designed to lift a 4600 kg package. What volume should the balloon have after being inflated with helium at 0°C and 1 atm pressure to lift the total load? (Hint: Use the density values in **Table 1.**)
- **4.** A submerged submarine alters its buoyancy so that it initially accelerates upward at 0.325 m/s<sup>2</sup>. What is the submarine's average density at this time? (Hint: the density of sea water is  $1.025 \times 10^3$  kg/m<sup>3</sup>.)
- **5. Critical Thinking** Many kayaks are made of plastics and other composite materials that are denser than water. How are such kayaks able to float in water?

# **SECTION 2**

#### SECTION OBJECTIVES

- Calculate the pressure exerted by a fluid.
- Calculate how pressure varies with depth in a fluid.

#### pressure

the magnitude of the force on a surface per unit area

#### **ADVANCED TOPICS**

See "Fluid Pressure" in Appendix J: Advanced Topics to learn more about other properties of fluids.



Figure 5 Atmospheric

Atmospheric diving suits allow divers to withstand the pressure exerted by the fluid in the ocean at depths of up to 610 m.

# **Fluid Pressure**

## PRESSURE

Deep-sea explorers wear atmospheric diving suits like the one shown in **Figure 5** to resist the forces exerted by water in the depths of the ocean. You experience the effects of similar forces on your ears when you dive to the bottom of a swimming pool, drive up a mountain, or ride in an airplane.

#### Pressure is force per unit area

In the examples above, the fluids exert **pressure** on your eardrums. Pressure is a measure of how much force is applied over a given area. It can be written as follows:

#### PRESSURE



The SI unit of pressure is the *pascal* (Pa), which is equal to  $1 \text{ N/m}^2$ . The pascal is a small unit of pressure. The pressure of the atmosphere at sea level is about  $1.01 \times 10^5$  Pa. This amount of air pressure under normal conditions is the basis for another unit, the *atmosphere* (atm). For the purpose of calculating pressure,  $10^5$  Pa is about the same as 1 atm. The absolute air pressure inside a typical automobile tire is about  $3 \times 10^5$  Pa, or 3 atm.

#### Applied pressure is transmitted equally throughout a fluid

When you pump a bicycle tire, you apply a force on the pump that in turn exerts a force on the air inside the tire. The air responds by pushing not only against the pump but also against the walls of the tire. As a result, the pressure increases by an equal amount throughout the tire.

In general, if the pressure in a fluid is increased at any point in a container (such as at the valve of the tire), the pressure increases at all points inside the container by exactly the same amount. Blaise Pascal (1623–1662) noted this fact in what is now called *Pascal's principle* (or *Pascal's law*):

#### PASCAL'S PRINCIPLE

Pressure applied to a fluid in a closed container is transmitted equally to every point of the fluid and to the walls of the container. A hydraulic lift, such as the one shown in **Figure 6**, makes use of Pascal's principle. A small force  $F_1$  applied to a small piston of area  $A_1$  causes a pressure increase in a fluid, such as oil. According to Pascal's principle, this increase in pressure,  $P_{inc}$ , is transmitted to a larger piston of area  $A_2$  and the fluid exerts a force  $F_2$  on this piston. Applying Pascal's principle and the definition of pressure gives the following equation:



Rearranging this equation to solve for  $F_2$  produces the following:

$$F_2 = \frac{A_2}{A_1} F_1$$

This second equation shows that the output force,  $F_2$  is larger than the input force,  $F_1$ , by a factor equal to the ratio of the areas of the two pistons. However, the input force must be applied over a longer distance; the work required to lift the truck is not reduced by the use of a hydraulic lift.

Figure 6 The press

 $A_1$ 

extension

"Hydraulic Lift Force."

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The pressure is the same on both sides of the enclosed fluid, allowing a small force to lift a heavy object.

## SAMPLE PROBLEM B

#### Pressure

#### **PROBLEM**

The small piston of a hydraulic lift has an area of 0.20 m<sup>2</sup>. A car weighing  $1.20 \times 10^4$  N sits on a rack mounted on the large piston. The large piston has an area of 0.90 m<sup>2</sup>. How large a force must be applied to the small piston to support the car?

 $F_2$ 

#### SOLUTION

Given:

$$A_1 = 0.20 \text{ m}^2$$
  $A_2 = 0.90 \text{ m}^2$   
 $F_2 = 1.20 \times 10^4 \text{ N}$ 

 $A_2$ 

#### Unknown:

Use the equation for pressure and apply Pascal's principle.

 $F_1 = ?$ 

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_1 = \left(\frac{A_1}{A_2}\right) F_2 = \left(\frac{0.20 \text{ m}^2}{0.90 \text{ m}^2}\right) (1.20 \times 10^4 \text{ N})$$

$$F_1 = 2.7 \times 10^3 \text{ N}$$

#### Pressure

- 1. In a car lift, compressed air exerts a force on a piston with a radius of 5.00 cm. This pressure is transmitted to a second piston with a radius of 15.0 cm.
  - **a.** How large a force must the compressed air exert to lift a  $1.33 \times 10^4$  N car?
  - **b.** What pressure produces this force? Neglect the weight of the pistons.
- **2.** A 1.5 m wide by 2.5 m long water bed weighs 1025 N. Find the pressure that the water bed exerts on the floor. Assume that the entire lower surface of the bed makes contact with the floor.
- **3.** A person rides up a lift to a mountaintop, but the person's ears fail to "pop"—that is, the pressure of the inner ear does not equalize with the outside atmosphere. The radius of each eardrum is 0.40 cm. The pressure of the atmosphere drops from  $1.010 \times 10^5$  Pa at the bottom of the lift to  $0.998 \times 10^5$  Pa at the top.
  - **a.** What is the pressure difference between the inner and outer ear at the top of the mountain?
  - **b.** What is the magnitude of the net force on each eardrum?



#### Pressure varies with depth in a fluid

As a submarine dives deeper in the water, the pressure of the water against the hull of the submarine increases, so the hull must be strong enough to withstand large pressures. Water pressure increases with depth because the water at a given depth must support the weight of the water above it.

Imagine a small area on the hull of a submarine. The weight of the entire column of water above that area exerts a force on the area. The column of water has a volume equal to *Ah*, where *A* is the cross-sectional area of the column and *h* is its height. Hence the mass of this column of water is  $m = \rho V = \rho Ah$ . Using the definitions of density and pressure, the pressure at this depth due to the weight of the column of water can be calculated as follows:

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{\rho Vg}{A} = \frac{\rho Ahg}{A} = \rho hg$$

This equation is valid only if the density is the same throughout the fluid.

The pressure in the equation above is referred to as *gauge pressure*. It is not the total pressure at this depth because the atmosphere itself also exerts a pressure at the surface. Thus, the gauge pressure is actually the total pressure minus the atmospheric pressure. By using the symbol  $P_0$  for the atmospheric pressure at the surface, we can express the total pressure, or *absolute pressure*, at a given depth in a fluid of uniform density  $\rho$  as follows:

#### FLUID PRESSURE AS A FUNCTION OF DEPTH

 $P = P_0 + \rho gh$ 

absolute pressure = atmospheric pressure + (density × free-fall acceleration × depth)

This expression for pressure in a fluid can be used to help understand buoyant forces. Consider a rectangular box submerged in a container of water, as shown in **Figure 7.** The water pressure at the top of the box is  $-(P_0 + \rho g h_1)$ , and the water pressure at the bottom of the box is  $P_0 + \rho g h_2$ . From the definition of pressure, we know that the downward force on the box is  $-A(P_0 + \rho g h_1)$ , where *A* is the area of the top of the box. The upward force on the box is  $A(P_0 + \rho g h_2)$ . The net force on the box is the sum of these two forces.

$$F_{net} = A(P_0 + \rho g h_2) - A(P_0 + \rho g h_1) = \rho g(h_2 - h_1)A = \rho g V = m_f g$$

Note that this is an expression of Archimedes' principle. In general, we can say that buoyant forces arise from the differences in fluid pressure between the top and the bottom of an immersed object.



#### Figure 7

The fluid pressure at the bottom of the box is greater than the fluid pressure at the top of the box.

#### -extension

#### **Practice Problems**

Visit <u>go.hrw.com</u> to find a sample and practice problems for pressure as a function of depth.



# SECTION REVIEW

- 1. Which of the following exerts the most pressure while resting on a floor?
  - a. a 25 N cube with 1.5 m sides
  - **b.** a 15 N cylinder with a base radius of 1.0 m
  - c. a 25 N cube with 2.0 m sides
  - d. a 25 N cylinder with a base radius of 1.0 m
- 2. Water is to be pumped to the top of the Empire State Building, which is 366 m high. What gauge pressure is needed in the water line at the base of the building to raise the water to this height? (Hint: See **Table 1** for the density of water.)
- **3.** When a submarine dives to a depth of  $5.0 \times 10^2$  m, how much pressure, in Pa, must its hull be able to withstand? How many times larger is this pressure than the pressure at the surface? (Hint: See **Table 1** for the density of sea water.)
- **4. Critical Thinking** Calculate the depth in the ocean at which the pressure is three times atmospheric pressure. (Hint: Use the value for the density of sea water given in **Table 1.**)

# **SECTION 3**

#### SECTION OBJECTIVES

- Examine the motion of a fluid using the continuity equation.
- Recognize the effects of Bernoulli's principle on fluid motion.



Figure 8

The water flowing around this cylinder exhibits laminar flow and turbulent flow.

#### ideal fluid

a fluid that has no internal friction or viscosity and is incompressible

# **Fluids in Motion**

# **FLUID FLOW**

Have you ever gone canoeing or rafting down a river? If so, you may have noticed that part of the river flowed smoothly, allowing you to float calmly or to simply paddle along. At other places in the river, there may have been rocks or dramatic bends that created foamy whitewater rapids.

When a fluid, such as river water, is in motion, the flow can be characterized in one of two ways. The flow is said to be *laminar* if every particle that

> passes a particular point moves along the same smooth path traveled by the particles that passed that point earlier. The smooth stretches of a river are regions of laminar flow.

> In contrast, the flow of a fluid becomes irregular, or *turbulent*, above a certain velocity or under conditions that can cause abrupt changes in velocity, such as where there are obstacles or sharp turns in a river. Irregular motions of the fluid, called *eddy currents*, are characteristic of turbulent flow.

**Figure 8** shows a photograph of water flowing past a cylinder. Hydrogen bubbles were added to the water to make the streamlines and the eddy currents visible. Notice the dramatic difference in flow patterns between the laminar flow and the turbulent flow. Laminar flow is much easier to model because it is predictable. Turbulent flow is extremely chaotic and unpredictable.

#### The ideal fluid model simplifies fluid-flow analysis

Many features of fluid motion can be understood by considering the behavior of an **ideal fluid.** Although no real fluid has all the properties of an ideal fluid, the ideal fluid model does help explain many properties of real fluids, so the model is a useful tool for analysis. While discussing density and buoyancy, we assumed all of the fluids used in problems were practically incompressible. A fluid is incompressible if the density of the fluid always remains constant.

The term *viscosity* refers to the amount of internal friction within a fluid. A fluid with a high viscosity flows more slowly than does a fluid with a low viscosity. As a viscous fluid flows, part of the kinetic energy of the fluid is transformed into internal energy because of the internal friction. Ideal fluids are considered *nonviscous*, so they lose no kinetic energy due to friction as they flow.

Ideal fluids are also characterized by a *steady flow*. In other words, the velocity, density, and pressure at each point in the fluid are constant. Ideal flow of an ideal fluid is also *nonturbulent*, which means that there are no eddy currents in the moving fluid.

# **PRINCIPLES OF FLUID FLOW**

Fluid behavior is often very complex. Several general principles describing the flow of fluids can be derived relatively easily from basic physical laws.

#### The continuity equation results from mass conservation

Imagine that an ideal fluid flows into one end of a pipe and out the other end, as shown in **Figure 9.** The diameter of the pipe is different at each end. How does the speed of fluid flow change as the fluid passes through the pipe?

Because mass is conserved and because the fluid is incompressible, we know that the mass flowing into the bottom of the pipe,  $m_1$ , must equal the mass flowing out of the top of the pipe,  $m_2$ , during any given time interval:

$$m_1 = m_2$$

This simple equation can be expanded by recalling that  $m = \rho V$  and by using the formula for the volume of a cylinder,  $V = A\Delta x$ .

$$\rho_1 V_1 = \rho_2 V_2$$
  
$$\rho_1 A_1 \Delta x_1 = \rho_2 A_2 \Delta x_2$$

The length of the cylinder,  $\Delta x$ , is also the distance the fluid travels, which is equal to the speed of flow multiplied by the time interval ( $\Delta x = v\Delta t$ ).

$$\rho_1 A_1 \nu_1 \Delta t = \rho_2 A_2 \nu_2 \Delta t$$

The time interval and, for an ideal fluid, the density are the same on each side of the equation, so they cancel each other out. The resulting equation is called the continuity equation:

**CONTINUITY EQUATION** 

$$A_1 \nu_1 = A_2 \nu_2$$

area  $\times$  speed in region 1 = area  $\times$  speed in region 2

#### The speed of fluid flow depends on cross-sectional area

Note in the continuity equation that  $A_1$  and  $A_2$  can represent any two different cross-sectional areas of the pipe, not just the ends. This equation implies that the fluid speed is faster where the pipe is narrow and slower where the pipe is wide. The product  $A\nu$ , which has units of volume per unit time, is called the *flow rate*. The flow rate is constant throughout the pipe.

The continuity equation explains an effect you may have observed as water flows slowly from a faucet, as shown in **Figure 10.** Because the water speeds up due to gravity as it falls, the stream narrows, satisfying the continuity equation. The continuity equation also explains why a river tends to flow more rapidly in places where the river is shallow or narrow than in places where the river is deep and wide.



#### Figure 9

The mass flowing into the pipe must equal the mass flowing out of the pipe in the same time interval.



Figure 10 The width of a stream of water narrows as the water falls and speed up.



#### Figure 11

A leaf speeds up as it passes into a constriction in a drainage pipe. The water pressure on the right is less than the pressure on the left.



#### Figure 12

As air flows around an airplane wing, the air above the wing moves faster than the air below, producing lift.

## The pressure in a fluid is related to the speed of flow

Suppose there is a water-logged leaf carried along by the water in a drainage pipe, as shown in **Figure 11.** The continuity equation shows that the water moves faster through the narrow part of the tube than through the wider part of the tube. Therefore, as the water carries the leaf into the constriction, the leaf speeds up.

If the water and the leaf are accelerating as they enter the constriction, an unbalanced force must be causing the acceleration, according to Newton's second law. This unbalanced force is a result of the fact that the water pressure in front of the leaf is less than the water pressure behind the leaf. The pressure difference causes the leaf and the water around it to accelerate as it enters the narrow part of the tube. This behavior illustrates a general principle known as *Bernoulli's principle*, which can be stated as follows:

#### **BERNOULLI'S PRINCIPLE**

#### The pressure in a fluid decreases as the fluid's velocity increases.

The lift on an airplane wing can be explained, in part, with Bernoulli's principle. As an airplane flies, air flows around the wings and body of the plane, as shown in **Figure 12.** Airplane wings are designed to direct the flow of air so that the air speed above the wing is greater than the air speed below the wing. Therefore, the air pressure above the wing is less than the pressure below, and there is a net upward force on the wing, called *lift*. The tilt of an airplane wing also adds to the lift on the plane. The front of the wing is tilted upward so that air striking the bottom of the wing is deflected downward.

# **SECTION REVIEW**

- 1. Water at a pressure of  $3.00 \times 10^5$  Pa flows through a horizontal pipe at a speed of 1.00 m/s. The pipe narrows to one-fourth its original diameter. What is the speed of the flow in the narrow section?
- **2.** A 2.0 cm diameter faucet tap fills a  $2.5 \times 10^{-2}$  m<sup>3</sup> container in 30.0 s. What is the speed at which the water leaves the faucet?
- **3. Critical Thinking** The time required to fill a glass with water from a large container with a spigot is 30.0 s. If you replace the spigot with a smaller one so that the speed of the water leaving the nozzle doubles, how long does it take to fill the glass?
- **4. Interpreting Graphics** For this problem, refer back to **Figure 9.** Assume that the cross-sectional area,  $A_2$ , in the tube is increased. Would the length,  $\Delta x_2$ , need to be longer or shorter for the mass of liquid in both sections to still be equal?

# **CHAPTER 8**

# Highlights

# **KEY IDEAS**

#### Section 1 Fluids and Buoyant Force

- Force is a vector quantity that causes changes in motion.
- A fluid is a material that can flow, and thus it has no definite shape. Both gases and liquids are fluids.
- Buoyant force is an upward force exerted by a fluid on an object floating on or submerged in the fluid.
- The magnitude of a buoyant force for a submerged object is determined by Archimedes' principle and is equal to the weight of the displaced fluid.
- The magnitude of a buoyant force for a floating object is equal to the weight of the object because the object is in equilibrium.

#### Section 2 Fluid Pressure

- Pressure is a measure of how much force is exerted over a given area.
- According to Pascal's principle, pressure applied to a fluid in a closed container is transmitted equally to every point of the fluid and to the walls of the container.
- The pressure in a fluid increases with depth.

#### Section 3 Fluids in Motion

- Moving fluids can exhibit laminar (smooth) flow or turbulent flow.
- An ideal fluid is incompressible, nonviscous, and, when undergoing ideal flow, nonturbulent.
- The continuity equation is derived from the fact that the amount of fluid leaving a pipe during some time interval equals the amount entering the pipe during that same time interval.
- According to Bernoulli's principle, swift-moving fluids exert less pressure than slower-moving fluids.

Variable Symbols			
antities	Units		Conversions
density	kg/m <sup>3</sup>	kilogram per meter <sup>3</sup>	$=10^{-3} \text{ g/cm}^{3}$
pressure	Ра	pascal	$= N/m^2$
	riable Symbols antities density pressure	riable SymbolsantitiesUnitsdensitykg/m³pressurePa	riable SymbolsmantitiesUnitsdensitykg/m³kilogram per meter³pressurePapascal

## **KEY TERMS**

fluid (p. 274) mass density (p. 275) buoyant force (p. 275) pressure (p. 280) ideal fluid (p. 284)

#### PROBLEM SOLVING

See **Appendix D: Equations** for a summary of the equations introduced in this chapter. If you need more problem-solving practice, see **Appendix I: Additional Problems.** 

# Review

# **DENSITY AND BUOYANCY**

## **Review Questions**

- 1. How is weight affected by buoyant force?
- 2. Buoyant force equals what for any floating object?

# **Conceptual Questions**

- **3.** If an inflated beach ball is placed beneath the surface of a pool of water and released, the ball shoots upward. Why?
- **4.** An ice cube is submerged in a glass of water. What happens to the level of the water as the ice melts?
- **5.** Will a ship ride higher in an inland freshwater lake or in the ocean? Why?
- **6.** Steel is much denser than water. How, then, do steel boats float?
- 7. A small piece of steel is tied to a block of wood. When the wood is placed in a tub of water with the steel on top, half of the block is submerged. If the block is inverted so that the steel is underwater, will the amount of the wooden block that is submerged increase, decrease, or remain the same?

## **Practice Problems**

#### For problems 8–9, see Sample Problem A.

**8.** An object weighs 315 N in air. When tied to a string, connected to a balance, and immersed in water, it weighs 265 N. When it is immersed in oil, it weighs 269 N. Find the following:

**a.** the density of the object

- **b.** the density of the oil
- **9.** A sample of an unknown material weighs 300.0 N in air and 200.0 N when submerged in an alcohol solution with a density of  $0.70 \times 10^3$  kg/m<sup>3</sup>. What is the density of the material?

# PRESSURE

#### **Review Questions**

- **10.** Is a large amount of pressure always caused by a large force? Explain your answer.
- **11.** What is the SI unit of pressure? What is it equal to, in terms of other SI units?

## Conceptual Questions

- **12.** After a long class, a physics teacher stretches out for a nap on a bed of nails. How is this possible?
- **13.** When drinking through a straw, you reduce the pressure in your mouth and the atmosphere moves the liquid. Could you use a straw to drink on the moon?

## **Practice Problems**

#### For problems 14–16, see Sample Problem B.

- 14. The four tires of an automobile are inflated to an absolute pressure of  $2.0 \times 10^5$  Pa. Each tire has an area of 0.024 m<sup>2</sup> in contact with the ground. Determine the weight of the automobile.
- **15.** A pipe contains water at  $5.00 \times 10^5$  Pa above atmospheric pressure. If you patch a 4.00 mm diameter hole in the pipe with a piece of bubble gum, how much force must the gum be able to withstand?
- **16.** A piston, *A*, as shown at right, has a diameter of 0.64 cm. A second piston, *B*, has a diameter of 3.8 cm. Determine the force, **F**, necessary to support the 500.0 N weight in the absence of friction.



# **FLUID FLOW**

## **Conceptual Questions**

- **17.** Prairie dogs live in underground burrows with at least two entrances. They ventilate their burrows by building a mound around one entrance, which is open to a stream of air. A second entrance at ground level is open to almost stagnant air. Use Bernoulli's principle to explain how this construction creates air flow through the burrow.
- **18.** Municipal water supplies are often provided by reservoirs built on high ground. Why does water from such a reservoir flow more rapidly out of a faucet on the ground floor of a building than out of an identical faucet on a higher floor?
- **19.** If air from a hair dryer is blown over the top of a table-tennis ball, the ball can be suspended in air. Explain how this suspension is possible.

#### **MIXED REVIEW**

- **20.** An engineer weighs a sample of mercury ( $\rho = 13.6 \times 10^3 \text{ kg/m}^3$ ) and finds that the weight of the sample is 4.5 N. What is the sample's volume?
- **21.** About how much force is exerted by the atmosphere on  $1.00 \text{ km}^2$  of land at sea level?
- **22.** A 70.0 kg man sits in a 5.0 kg chair so that his weight is evenly distributed on the legs of the chair. Assume that each leg makes contact with the floor over a circular area with a radius of 1.0 cm. What is the pressure exerted on the floor by each leg?
- **23.** A frog in a hemispherical bowl, as shown below, just floats in a fluid with a density of  $1.35 \times 10^3$  kg/m<sup>3</sup>. If the bowl has a radius of 6.00 cm and negligible mass, what is the mass of the frog?



- **24.** When a load of  $1.0 \times 10^6$  N is placed on a battleship, the ship sinks only 2.5 cm in the water. Estimate the cross-sectional area of the ship at water level. (Hint: See **Table 1** for the density of sea water.)
- **25.** A 1.0 kg beaker containing 2.0 kg of oil with a density of 916 kg/m<sup>3</sup> rests on a scale. A 2.0 kg block of iron is suspended from a spring scale and completely submerged in the oil, as shown at right. Find the equilibrium readings of both scales. (Hint: See **Table 1** for the density of iron.)



- **26.** A raft is constructed of wood having a density of 600.0 kg/m<sup>3</sup>. The surface area of the bottom of the raft is 5.7 m<sup>2</sup>, and the volume of the raft is 0.60 m<sup>3</sup>. When the raft is placed in fresh water having a density of  $1.0 \times 10^3$  kg/m<sup>3</sup>, how deep is the bottom of the raft below water level?
- **27.** A physics book has a height of 26 cm, a width of 21 cm, and a thickness of 3.5 cm.
  - **a.** What is the density of the physics book if it weighs 19 N?
  - **b.** Find the pressure that the physics book exerts on a desktop when the book lies face up.
  - **c.** Find the pressure that the physics book exerts on the surface of a desktop when the book is balanced on its spine.
- **28.** A natural-gas pipeline with a diameter of 0.250 m delivers 1.55 m<sup>3</sup> of gas per second. What is the flow speed of the gas?
- **29.** A 2.0 cm thick bar of soap is floating in water, with 1.5 cm of the bar underwater. Bath oil with a density of 900.0 kg/m<sup>3</sup> is added and floats on top of the water. How high on the side of the bar will the oil reach when the soap is floating in only the oil?
- **30.** Which dam must be stronger, one that holds back  $1.0 \times 10^5$  m<sup>3</sup> of water 10 m deep or one that holds back  $1.0 \times 10^3$  m<sup>3</sup> of water 20 m deep?

**31.** A light spring with a spring constant of 90.0 N/m rests vertically on a table, as shown in (a) below. A 2.00 g balloon is filled with helium (0°C and 1 atm pressure) to a volume of 5.00 m<sup>3</sup> and connected to the spring, causing the spring to stretch, as shown

in (b) at right. How much does the spring stretch when the system is in equilibrium? (Hint: See **Table 1** for the density of helium. The magnitude of the spring force equals  $k\Delta x$ .)



- **32.** The aorta in an average adult has a cross-sectional area of  $2.0 \text{ cm}^2$ .
  - **a.** Calculate the flow rate (in grams per second) of blood ( $\rho = 1.0 \text{ g/cm}^3$ ) in the aorta if the flow speed is 42 cm/s.
  - **b.** Assume that the aorta branches to form a large number of capillaries with a combined cross-sectional area of  $3.0 \times 10^3$  cm<sup>2</sup>. What is the flow speed in the capillaries?
- **33.** A 1.0 kg hollow ball with a radius of 0.10 m is filled with air and is released from rest at the bottom of a 2.0 m deep pool of water. How high above the surface of the water does the ball rise? Disregard friction and the ball's motion when the ball is only partially submerged.



#### **Flow Rates**

Flow rate, as you learned earlier in this chapter, is described by the following equation:

#### flow rate = Av

Flow rate is a measure of the volume of a fluid that passes through a tube per unit time. A is the crosssectional area of the tube, and v is the flow speed of the fluid. If A has units of centimeters squared and v has units of centimeters per second, flow rate will have units of cubic centimeters per second.

The graphing calculator will use the following equation to determine flow rate.

$$Y_1 = \pi^* V(X/2)^2$$

You will use this equation to study the flow rates  $(Y_1)$  for various hose diameters (X) and flow speeds (V). The calculator will produce a table of flow rates in cubic centimeters per second versus hose diameters in centimeters.

In this graphing calculator activity, you will learn how to read a table on the calculator and to use that table to make predictions about flow rates.

Visit <u>go.hrw.com</u> and enter the keyword **HF6FLUX** to find this graphing calculator activity. Refer to **Appendix B** for instructions on downloading the program for this activity.

- **34.** In testing a new material for shielding spacecraft, 150 ball bearings each moving at a supersonic speed of 400.0 m/s collide head-on and elastically with the material during a 1.00 min interval. If the ball bearings each have a mass of 8.0 g and the area of the tested material is 0.75 m<sup>2</sup>, what is the pressure exerted on the material?
- **35.** A thin, rigid, spherical shell with a mass of 4.00 kg and diameter of 0.200 m is filled with helium (adding negligible mass) at 0°C and 1 atm pressure. It is then released from rest on the bottom of a pool of water that is 4.00 m deep.
  - **a.** Determine the upward acceleration of the shell.
  - **b.** How long will it take for the top of the shell to reach the surface? Disregard frictional effects.
- **36.** A student claims that if the strength of Earth's gravity doubled, people would be unable to float on water. Do you agree or disagree with this statement? Why?

**37.** A light spring with a spring constant of 16.0 N/m rests vertically on the bottom of a large beaker of water, as shown in (**a**) below. A  $5.00 \times 10^{-3}$  kg block of wood with a density of 650.0 kg/m<sup>3</sup> is connected to the spring, and the mass-spring system is allowed to come to static equilibrium, as shown in (**b**) below. How much does the spring stretch?



- **38.** Astronauts sometimes train underwater to simulate conditions in space. Explain why.
- **39.** Explain why balloonists use helium instead of air in balloons.

# Alternative Assessment

- 1. Build a hydrometer from a long test tube with some sand at the bottom and a stopper. Adjust the amount of sand as needed so that the tube floats in most liquids. Calibrate it, and place a label with markings on the tube. Measure the densities of the following liquid foods: skim milk, whole milk, vegetable oil, pancake syrup, and molasses. Summarize your findings in a chart or table.
- 2. The owner of a fleet of tractor-trailers has contacted you after a series of accidents involving tractortrailers passing each other on the highway. The owner wants to know how drivers can minimize the pull exerted as one tractor-trailer passes another going in the same direction. Should the passing tractor-trailer try to pass as quickly as possible or as

slowly as possible? Design experiments to determine the answer by using model motor boats in a swimming pool. Indicate exactly what you will measure and how. If your teacher approves your plan and you are able to locate the necessary equipment, perform the experiment.

**3.** Record any examples of pumps in the tools, machines, and appliances you encounter in one week, and briefly describe the appearance and function of each pump. Research how one of these pumps works, and evaluate the explanation of the pump's operation for strengths and weaknesses. Share your findings in a group meeting and create a presentation, model, or diagram that summarizes the group's findings.



# Standardized Test Prep

# **MULTIPLE CHOICE**

- **1.** Which of the following is the correct equation for the net force acting on a submerged object?
  - **A.**  $F_{net} = 0$
  - **B.**  $F_{net} = (\rho_{object} \rho_{fluid})gV_{object}$
  - **C.**  $F_{net} = (\rho_{fluid} \rho_{object})gV_{object}$
  - **D.**  $F_{net} = (\rho_{fluid} + \rho_{object})gV_{object}$
- 2. How many times greater than the lifting force must the force applied to a hydraulic lift be if the ratio of the area where pressure is applied to the lifted area is  $\frac{1}{7}$ ?

**F.** 
$$\frac{1}{49}$$
  
**G.**  $\frac{1}{7}$   
**H.** 7  
**J.** 49

- **3.** A typical silo on a farm has many bands wrapped around its perimeter, as shown in the figure below. Why is the spacing between successive bands smaller toward the bottom?
  - **A.** to provide support for the silo's sides above them
  - **B.** to resist the increasing pressure that the grains exert with increasing depth
  - **C.** to resist the increasing pressure that the atmosphere exerts with increasing depth
  - **D.** to make access to smaller quantities of grain near the ground possible



- **4.** A fish rests on the bottom of a bucket of water while the bucket is being weighed. When the fish begins to swim around in the bucket, how does the reading on the scale change?
  - **F.** The motion of the fish causes the scale reading to increase.
  - **G.** The motion of the fish causes the scale reading to decrease.
  - **H.** The buoyant force on the fish is exerted downward on the bucket, causing the scale reading to increase.
  - **J.** The mass of the system, and so the scale reading, will remain unchanged.

# Use the passage below to answer questions 5–6.

A metal block ( $\rho = 7900 \text{ kg/m}^3$ ) is connected to a spring scale by a string 5 cm in length. The block's weight in air is recorded. A second reading is recorded when the block is placed in a tank of fluid and the surface of the fluid is 3 cm below the scale.

- **5.** If the fluid is oil ( $\rho < 1000 \text{ kg/m}^3$ ), which of the following must be true?
  - **A.** The first scale reading is larger than the second reading.
  - **B.** The second scale reading is larger than the first reading.
  - **C.** The two scale readings are identical.
  - **D.** The second scale reading is zero.
- **6.** If the fluid is mercury ( $\rho = 13600 \text{ kg/m}^3$ ), which of the following must be true?
  - **F.** The first scale reading is larger than the second reading.
  - **G.** The second scale reading is larger than the first reading.
  - H. The two scale readings are identical.
  - **J.** The second scale reading is zero.

#### Use the passage below to answer questions 7–8.

Water flows through a pipe of varying width at a constant mass flow rate. At point *A* the diameter of the pipe is  $d_A$  and at point *B* the diameter of the pipe is  $d_B$ .

7. Which of the following equations describes the relationship between the water speed at point *A*, *ν*<sub>A</sub>, and the water speed at point *B*, *ν*<sub>A</sub>?

**A.** 
$$d_A \nu_A = d_B \nu_B$$
  
**B.**  $d_A^2 \nu_A = d_B^2 \nu_B$ 

- **C.**  $d_A d_B = v_A v_B$ **D.**  $\frac{1}{2} d_A v_A^2 = \frac{1}{2} d_B v_B^2$
- 8. If the cross-sectional area of point A is 2.5 m<sup>2</sup> and the cross-sectional area of point B is 5.0 m<sup>2</sup>, how many times faster does the water flow at point A than at point B?
  - **F.**  $\frac{1}{4}$ **G.**  $\frac{1}{2}$ **H.** 2 **J.** 4

# **SHORT RESPONSE**

- **9.** Will an ice cube float higher in water or in mercury? Explain your answer.
- 10. The approximate inside diameter of the aorta is 1.6 cm, and that of a capillary is  $1.0 \times 10^{-6} \text{ m}$ . The average flow speed is about 1.0 m/s in the aorta and 1.0 cm/s in the capillaries. If all the blood in the aorta eventually flows through the capillaries, estimate the number of capillaries.

11. A hydraulic brake system is shown below. The area of the piston in the master cylinder is  $6.40 \text{ cm}^2$ , and the area of the piston in the brake cylinder is  $1.75 \text{ cm}^2$ . The coefficient of friction between the brake shoe and wheel drum is 0.50. What is the frictional force between the brake shoe and wheel drum when a force of 44 N is exerted on the pedal?



# **EXTENDED RESPONSE**

#### Base your answers to questions 12–14 on the information below.

Oil, which has a density of 930.0 kg/m<sup>3</sup>, floats on water. A rectangular block of wood with a height, h, of 4.00 cm and a density of 960.0 kg/m<sup>3</sup> floats partly in the water, and the rest floats completely under the oil layer.

- **12.** What is the balanced force equation for this situation?
- **13.** What is the equation that describes *y*, the thickness of the part of the block that is submerged in water?
- **14.** What is the value for *y*?

**Test TIP** For problems involving several forces, write down equations showing how the forces interact.

# **Physics and Its World** *Timeline* 1690–1785

**1698** – The Ashanti empire, the last of the major African kingdoms, emerges in what is now Ghana. The Ashanti's strong centralized government and effective bureaucracy enable them to control the region for nearly two centuries.



Thomas Newcomen invents the first practical steam engine. Over 50 years later, James Watt makes significant improvements to the Newcomen engine.



**1715** (approx.) – Chinese writer **Ts'ao Hsüeh-ch'in** is born. The book *The Dream of the Red Chamber*, attributed to him and another writer, is widely regarded today as the greatest Chinese novel.



1721 – Johann Sebastian Bach completes the six Brandenburg Concertos.



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**1735** – John Harrison constructs the first of four chronometers that will allow navigators to accurately determine a ship's longitude.



**1738** – Under the leadership of **Nadir Shah**, the Persian Empire expands into India as the Moghul Empire enters a stage of decline.

1738

 $P + \frac{1}{2}\rho\nu^2 + \rho gh = \text{constant}$ 

**Daniel Bernoulli's** *Hydrodynamics*, which includes his research on the mechanical behavior of fluids, is published.

#### 1752



Benjamin Franklin performs the dangerous "kite experiment," in which he demonstrates that lightning consists of electric charge. He would build on the first studies of electricity performed earlier in the century by describing electricity as having positive and negative charge.

1747 - Contrary to the favored idea that heat is a fluid, Russian chemist Mikhail V. Lomonosov publishes his hypothesis that heat is the result of motion. Several years later, Lomonosov formulates conservation laws for mass and energy.





Laurent Lavoisier begins his research on chemical reactions, notably oxidation and combustion.

1756 - The Seven Year's War begins. British general James Wolfe leads the capture of Fort Louisburg, in Canada, in 1758.

1770 - Antoine



1772 - Caroline Herschel, sister of astronomer William Herschel, joins her brother in England. She compiles the most comprehensive star catalog of the era and discovers several nebulae-regions of glowing gas-within our galaxy.





1775 - The American Revolution begins.

Physics and Its World 1690-1785

#### 1785

 $F_{electric} = k_C \left( \frac{q_1 q_2}{r^2} \right)$ 

Charles Augustin de Coulomb publishes the results of experiments that will systematically and conclusively prove the inverse-square law for electric force. The law has been suggested for over 30 years by other scientists, such as Daniel Bernoulli, Joseph Priestly, and Henry Cavendish.



Herschel emigrates to England to avoid fighting in the Seven Year's War. Over the next 60 years, he pursues astronomy, constructing the largest reflecting telescopes of the era and discovering new objects, such as binary stars and the planet Uranus.

1757 – German musician William

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